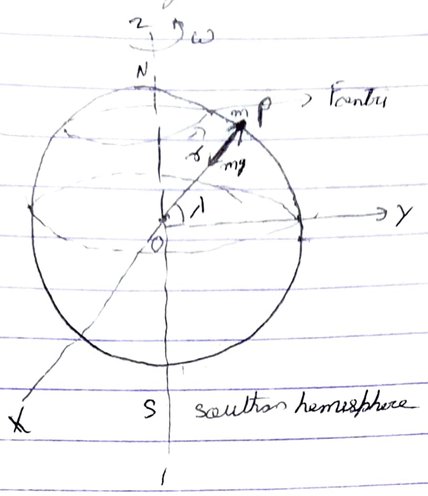
B.Sc - I

29/1/22

Effect of rotation of earth on accⁿ due to gravity or variation of g' with lattitude λ



Fictitious force

(1) $-m(\vec{\omega}\times\vec{r})$ → transverse

(2) $-2m(\vec{\omega}\times\vec{v})$ → coriolis force

The various forces on particle of mass m at P

(i) Real force $m\vec{g} = -mg\hat{r}$ — (1)

(ii) Fictitious force $\vec{F}_{centri} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r})$ — (2)

$$\vec{\omega} = \omega\hat{k}$$

$$\vec{r} = r\cos\lambda\hat{j} + r\sin\lambda\hat{k}$$

$$\vec{F}_{cent.} = -m\omega\hat{k}\times(\omega\hat{k}\times(r\cos\lambda\hat{j}+r\sin\lambda\hat{k}))$$

$$= -m\omega\hat{k}\times(r\omega\cos\lambda(-\hat{i})).$$

$$= +mr\omega^2\cos\lambda(\hat{k}\times\hat{i})$$

$$= mr\omega^2\cos\lambda\hat{j} \quad \text{— (3)}$$

The effective weight of particle at P

$$m\vec{g}' = m\vec{g} + \vec{F}_{centrifugal}$$

$$= -mg\hat{r} + mr\omega^2\cos\lambda\hat{j} \quad \text{— (4)}$$

$$\cancel{m}\vec{g}' = -\cancel{m}g(\cos\lambda\hat{j}+\sin\lambda\hat{k}) + m\omega^2 r\cos\lambda\hat{j}$$

$$= (\omega^2 r\cos\lambda - g\cos\lambda)\hat{j} - g\sin\lambda\hat{k}$$

$\therefore$ magnitude of $\vec{g}'$

$$g' = \left[(\omega^2 r\cos\lambda - g\cos\lambda)^2 + g^2\sin^2\lambda\right]^{1/2}$$

$$g' = \left(\omega^4 r^2\cos^2\lambda + g^2\cos^2\lambda - 2\omega^2 r\cos^2\lambda\, g + g^2\sin^2\lambda\right)^{1/2}$$

$$g' = \left[g^2 + \omega^4 r^2\cos^2\lambda - 2\omega^2 r\cos^2\lambda\, g\right]^{1/2}$$

$$g' = g\left[1 + \underbrace{\frac{\omega^4 r^2\cos^2\lambda}{g^2}}_{\text{Its contribution is very small}} - \frac{2\omega^2 r\cos^2\lambda}{g}\right]^{1/2}$$

$$g' = g\left(1 - \frac{2\omega^2 r\cos^2\lambda}{g}\right)^{1/2}$$

$$\cancel{g' = g(1 - \ldots)}$$

$$g' = g\left(1 - \frac{1}{2}\,\frac{2\omega^2 r\cos^2\lambda}{g}\right)$$

$$\boxed{g' = g - \omega^2 r\cos^2\lambda}$$

Special case

(i) At equator $\lambda = 0°$ $\cos^2\lambda = 1$

$$g' = g - \omega^2 r$$

(ii) At Pole $\lambda = 90°$

$$\cos^2\lambda = 0$$

$$g' = g = 9.8\ m/s^2$$

That is no effect of g at poles due to rotation of earth.

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